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## ALGEBRA.

129. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, O.

$$\left| \begin{array}{cccccc} 1 - \binom{m-1}{0} - \binom{m-1}{1} \dots \binom{m-1}{m-2} \\ 1 & 1 - \binom{m-2}{0} \dots \binom{m-2}{m-3} \\ 1 & 0 & 1 & \dots \binom{m-3}{m-4} \\ \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 1 & \binom{1}{0} \\ 1 & 0 & 0 \dots 0 & 1 \end{array} \right| e = \sum_{n=1}^{\infty} \frac{n^m}{n!} \text{, where } \binom{m-2}{k} = \frac{(m-2) \dots (m-k-1)}{k!}$$

Solution by the PROPOSER.

$$\text{Let } f(x) = e^{ex} = 1 + e^x + \frac{e^{2x}}{2!} + \frac{e^{3x}}{3!} + \frac{e^{4x}}{4!} + \frac{e^{5x}}{5!} + \dots + \frac{e^{rx}}{r!} + \dots$$

$$= 1 + (1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots)$$

$$+ \frac{1}{2!} (1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!} + \dots)$$

$$+ \frac{1}{3!} (1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots)$$

.....

$$+ \frac{1}{r!} (1 + rx + \frac{(rx)^2}{2!} + \frac{(rx)^3}{3!} + \dots)$$

.....

$$= e + \sum_{r=1}^{r=\infty} a_r x^r, \text{ where } a_r = \frac{1}{r!} \sum_{k=1}^{k=\infty} \frac{k^r}{k!}.$$

$$\text{But } f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \dots$$

$$\therefore f^m(0) = \sum_{k=1}^{k=\infty} \frac{k^m}{k!} \dots (1).$$

Let  $y = e^{ex}$ , then  $\log y = ex$ .

$\therefore \frac{dy}{dx} = ye^x$ . And by Leibnitz's Formula,

$$f^m(0) = f^{m-1}(0) + (m-1)f^{m-2}(0) + \frac{(m-1)(m-2)}{2!}f^{m-3}(0) + \dots$$

$$+ (m-1)f'(0) + f(0) = \sum_{k=1}^{k=\infty} \frac{k^m}{k!} \cdot [ \text{by (1)} ] \quad \dots (2).$$

Or, letting  $x_m$  represent  $f^m(0)$ , etc.,

$$x_m - x_{m-1} - C_{m-1,1}x_{m-2} - \dots - C_{m-1,1}x_1 = f(0) = e.$$

$$x_{m-1} - x_{m-2} - C_{m-2,1}x_{m-3} - \dots - C_{m-2,1}x_1 = e$$

.....

.....

$$x_2 - x_1 = e$$

$$x_1 = e$$

$$\therefore x_m = \begin{vmatrix} e & -\binom{m-1}{0} & -\binom{m-1}{1} & \dots & -\binom{m-1}{m-2} \\ e & 1 & -\binom{m-2}{0} & \dots & -\binom{m-2}{m-3} \\ e & 0 & 1 & \dots & -\binom{m-3}{m-4} \\ e & 0 & 0 & 1 & \dots & -\binom{m-4}{m-5} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ e & 0 & 0 & 0 & \dots & 1 & -\binom{1}{0} \\ e & 0 & 0 & 0 & \dots & 0 & 1 \end{vmatrix}$$

$$\therefore \begin{vmatrix} 1 & -\binom{m-1}{0} & -\binom{m-1}{1} & \dots & -\binom{m-1}{m-2} \\ 1 & 1 & -\binom{m-2}{0} & \dots & -\binom{m-2}{m-3} \\ 1 & 0 & 1 & \dots & -\binom{m-3}{m-4} \\ 1 & 0 & 0 & 1 & \dots & -\binom{m-4}{m-5} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 0 & \dots & 1 & -\binom{1}{0} \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{vmatrix} e = \sum_{k=1}^{k=\infty} \frac{k^m}{k!} \dots (3).$$

By using (2) or (3) the following special relations have been obtained :

$$2e = \sum_{k=1}^{k=\infty} \frac{k^2}{k!}, \quad 5e = \sum_{k=1}^{k=\infty} \frac{k^3}{k!}, \quad 15e = \sum_{k=1}^{k=\infty} \frac{k^4}{k!}, \quad 52e = \sum_{k=1}^{k=\infty} \frac{k^5}{k!}, \quad 203e = \sum_{k=1}^{k=\infty} \frac{k^6}{k!}, \quad 877e = \sum_{k=1}^{k=\infty} \frac{k^7}{k!},$$

$$4140e = \sum_{k=1}^{k=\infty} \frac{k^8}{k!}, \quad 21147e = \sum_{k=1}^{k=\infty} \frac{k^9}{k!}, \quad 115975e = \sum_{k=1}^{k=\infty} \frac{k^{10}}{k!}.$$

Also solved by **G. B. M. ZERR**.

### GEOMETRY.

160. Proposed by **G. B. M. ZERR**, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let  $GFH$  be the spherical triangle formed by joining the mid-points of the sides of the spherical triangle  $ABC$ ;  $E$  the spherical excess of  $ABC$ ;  $\beta, p$  the base and altitude of  $GFH$ . Prove  $\sin \frac{1}{2}E = \sin \beta \sin p$ .

Solution by **G. B. M. ZERR**, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let  $BA=c$ ,  $GF=\gamma$ ,  $FH=\beta$ ,  $GH=\delta$ ,  $GP=-p$ .

Draw  $AL$ ,  $BM$ ,  $CK$  perpendicular to  $DGFC$ . Now  $DABE=DGFE=\pi$ .

$AD=BE=\frac{1}{2}(\pi-c)$ ,  $DL=ME$ ,  $LG=CK$ ,  $KF=FM$ .

$\therefore 2(DL+GK+KF)=\pi$ . Also  $2GK+2KF=2\gamma$ .

$\therefore 2DL+2\gamma=\pi$ , or  $DL=\frac{1}{2}\pi-\gamma$ .

$\angle DAL=\angle EBM$ ,  $\angle LAG=\angle GCK$ ,  $\angle KCF=\angle FBM$ .

$\therefore 2\angle DAL+C+A+B=2\pi$ .

$\therefore \angle DAL=\pi-\frac{1}{2}(A+B+C)=\pi-s=\frac{1}{2}(\pi-E)$ .

$\cos DAL=\sin D \cos DL$ .  $\cos \frac{1}{2}(\pi-E)=\sin D \cos (\frac{1}{2}\pi-\gamma)$ .

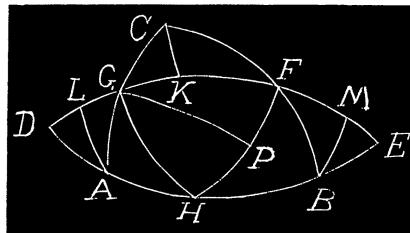
$\therefore \sin \frac{1}{2}E=\sin D \sin \gamma$ .

Now  $\sin D : \sin DFH = \sin \beta : \sin DH$ . But  $DH=\frac{1}{2}\pi$ .

$\therefore \sin D = \sin \beta \sin DFH$ . But  $\sin p = \sin \gamma \sin DFH$ .

$\therefore \sin D = (\sin \beta \sin p) / \sin \gamma$ .

$\therefore \sin \frac{1}{2}E = \sin \beta \sin p$ .



Also solved by **J. SCHEFFER** and **L. C. WALKER**.

161. Proposed by **MARCUS BAKER**, U. S. Coast and Geodetic Survey Office, Washington, D. C.

A circle, radius  $r$ , is inscribed in a triangle  $ABC$ . In the angles  $A$ ,  $B$ , and  $C$  are inscribed circles each touching two sides and the inscribed circle. There are six such circles. The first group of three have their centers between the incenters and the vertices, and the second group of three does not. Let  $r_a$ ,  $r_b$ ,  $r_c$  denote the radii of the first group. Then this well known relation holds:  $r = \sqrt{r_a r_b} + \sqrt{r_b r_c} + \sqrt{r_c r_a}$ . Let  $R_a$ ,  $R_b$ ,  $R_c$  denote the radii of the second group. Then this relation holds: